

Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
NUMBER

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

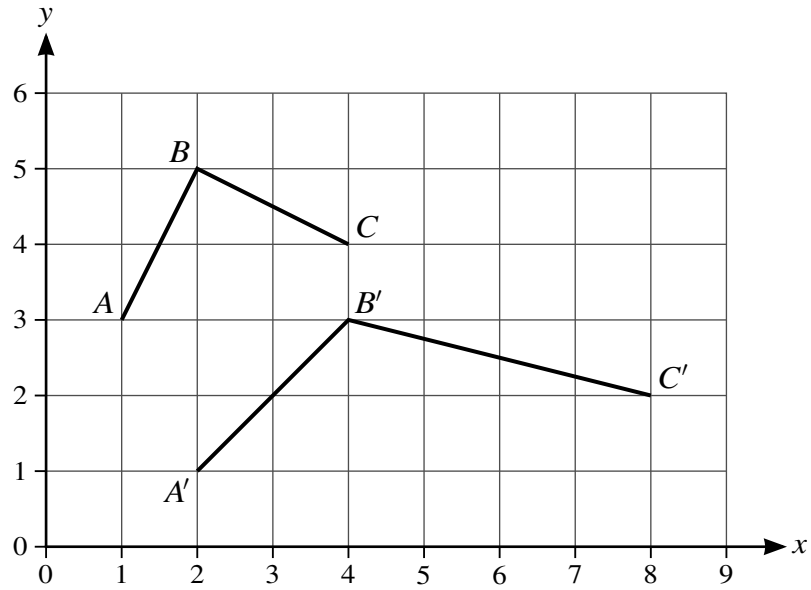
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



The diagram shows the graph of $y = f(x)$, which consists of the two straight lines AB and BC . The lines $A'B'$ and $B'C'$ form the graph of $y = g(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = f(x)$.

State fully the two transformations.

[4]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

- 2** The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 - 6x + c$, where c is a constant. It is given that $f(x) > 2$ for all values of x .

Find the set of possible values of c .

[4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3 (a)** Give the complete expansion of $\left(x + \frac{2}{x}\right)^5$. [2]

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- (b)** In the expansion of $(a + bx^2)\left(x + \frac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\frac{1}{x}$ is 80.

Find the values of the constants a and b . [4]

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- 4 (a) Show that the equation

$$3 \tan^2 x - 3 \sin^2 x - 4 = 0$$

may be expressed in the form $a \cos^4 x + b \cos^2 x + c = 0$, where a , b and c are constants to be found. [3]

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- (b) Hence solve the equation $3 \tan^2 x - 3 \sin^2 x - 4 = 0$ for $0^\circ \leq x \leq 180^\circ$. [4]

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- 5 A circle has equation $(x - 1)^2 + (y + 4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B .

(a) Find the coordinates of the two points of intersection. [4]

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(b) Find an equation of the circle with diameter AB . [3]

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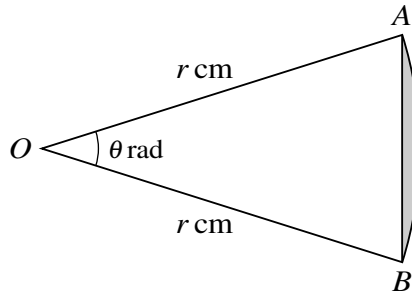
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The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is 76.8 cm^2 .

- (a) Find the area of the shaded region. [5]

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- (b) Find the perimeter of the shaded region. [2]

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7 The function f is defined by $f(x) = 2 - \frac{5}{x+2}$ for $x > -2$.

(a) State the range of f .

[1]

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

[4]

[illegible]

The function g is defined by $g(x) = x + 3$ for $x > 0$.

(c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax+b}{cx+d}$, where a , b , c and d are integers. [3]

[illegible]

- (a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480 . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 9 A curve which passes through $(0, 3)$ has equation $y = f(x)$. It is given that $f'(x) = 1 - \frac{2}{(x-1)^3}$.

(a) Find the equation of the curve.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

The tangent to the curve at $(0, 3)$ intersects the curve again at one other point, P .

- (b) Show that the x -coordinate of P satisfies the equation $(2x + 1)(x - 1)^2 - 1 = 0$. [4]

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- (c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y -coordinate of P . [2]

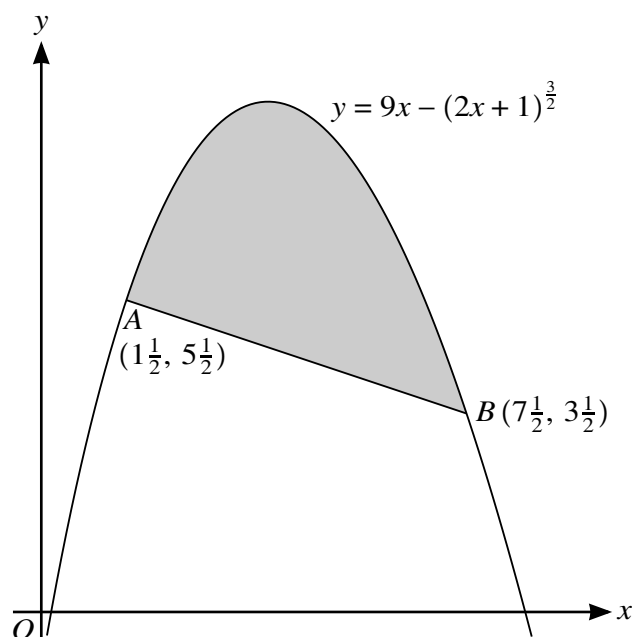
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The diagram shows the points $A(1\frac{1}{2}, 5\frac{1}{2})$ and $B(7\frac{1}{2}, 3\frac{1}{2})$ lying on the curve with equation $y = 9x - (2x + 1)^{\frac{3}{2}}$.

- (a) Find the coordinates of the maximum point of the curve. [4]

[illegible]

- (b) Verify that the line AB is the normal to the curve at A . [3]

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- (c) Find the area of the shaded region. [5]

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[illegible]

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